Optimal magnetic Sobolev constants in the semiclassical limit

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Abstract: We introduce the following nonlinear eigenvalue, or optimal magnetic Sobolev constant:

$$\lambda(\Omega, \mathbf{A}, p, h) = \inf_{\psi \in H_0^1(\Omega), \psi \neq 0} \frac{\mathcal{Q}_{h,\mathbf{A}}(\psi)}{\left(\int_\Omega |\psi|^p dx\right)^{\frac{2}{p}}} = \inf_{\substack{\psi \in H_0^1(\Omega), \\ \|\psi\|_{L^p(\Omega)}=1}} \mathcal{Q}_{h,\mathbf{A}}(\psi), \tag{0.1}$$

where the magnetic quadratic form is defined by

$$\forall \psi \in H_0^1(\Omega), \quad \mathcal{Q}_{h,\mathbf{A}}(\psi) = \int_\Omega |(-ih\nabla + \mathbf{A})\psi|^2 dx.$$

This object, and the corresponding minimizing functions, are of obvious interest in nonlinear evolution problems.

We obtain—under different classes of assumptions on the magnetic field generated by the vector potential $\mathbf{A}$—leading order asymptotic estimates on $\lambda(\Omega, \mathbf{A}, p, h)$ as well as localisation estimates for the minimizers.

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