Plane-like minimizers for a non-local Ginzburg-Landau-type energy in a periodic medium

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We consider a non-local energy of the form

$$\mathscr{E}(u) := \frac{1}{2} \int_{\mathbb{R}^n} \int_{\mathbb{R}^n} |u(x) - u(y)|^2 \, K(x,y) \, dx dy + \int_{\mathbb{R}^n} W(x, u(x)) \, dx,$$

where $K : \mathbb{R}^n \times \mathbb{R}^n \to [0, +\infty]$ is a measurable kernel comparable to that of the fractional Laplacian of order $2s$, with $s \in (0,1)$, and $W : \mathbb{R}^n \times \mathbb{R} \to [0, +\infty)$ is a smooth double-well potential, with zeroes at $u = \pm 1$. Both K and W are assumed to be $\mathbb{Z}^n$-periodic.

For any vector $\omega \in \mathbb{R}^n \setminus \{0\}$, we prove the existence of a minimizer u_ω of $\mathscr{E}$ that is *directed* along ω and whose interface $\{|u_\omega| < 9/10\}$ is contained in a strip $\{\omega \cdot x \in [0, M_0|\omega|]\}$ of universal width $M_0 > 0$. Moreover, u_ω enjoys a suitable periodicity/almost-periodicity property, in dependence of whether ω is rational or not.

As a result, we obtain the existence of *plane-like* entire solutions to the integro-differential Euler-Lagrange equation corresponding to $\mathscr{E}$.

This is a joint work with Prof. E. Valdinoci (WIAS, Berlin).