SIGN CHANGING SOLUTIONS TO A BAHRI-CORON’S PROBLEM IN PIERCED DOMAINS

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Abstract. We consider the problem
\[
\begin{aligned}
-\Delta u &= |u|^{\frac{4}{N-2}}u \quad \text{in } \Omega \setminus \{B(\xi_1, \varepsilon) \cup B(\xi_2, \varepsilon)\}, \\
\quad u &= 0 \quad \text{on } \partial (\Omega \setminus \{B(\xi_1, \varepsilon) \cup B(\xi_2, \varepsilon)\}),
\end{aligned}
\]
where \( \Omega \) is a smooth bounded domain in \( \mathbb{R}^N \), \( N \geq 3 \), \( \xi_1, \xi_2 \) are different points in \( \Omega \) and \( \varepsilon \) is a small positive parameter. We show that, for \( \varepsilon \) small enough, the equation has at least one pair of sign changing solutions, whose positive and negative parts concentrate at \( \xi_1 \) and \( \xi_2 \) as \( \varepsilon \) goes to zero.

1. Introduction. Let \( D \) be a smooth bounded domain in \( \mathbb{R}^N \), \( N \geq 3 \). Consider the following nonlinear elliptic problem
\[
\Delta u + |u|^{\frac{4}{N-2}}u = 0 \quad \text{in } D, \quad u = 0 \quad \text{on } \partial D.
\]
It is well known that the Sobolev embedding \( H^1_0(D) \hookrightarrow L^{\frac{2N}{N-2}}(D) \) is not compact and that this lack of compactness makes the question of solvability of (1) quite delicate.

Pohozaev’s identity [31] shows that problem (1) has only the trivial solution if the domain \( D \) is assumed to be strictly starshaped. On the other hand, Kazdan and Warner showed in [23] that if \( D \) is an annulus then (1) has a (unique) positive solution in the class of functions with radial symmetry. In [7], the authors study the asymptotic behavior of this solution as the radius of the inner ball of the annulus tends to zero. In the nonsymmetric case, Coron [17] found via variational methods that (1) is solvable and that it admits a positive solution under the assumption that \( D \) is a domain exhibiting a small hole. Substantial improvement of this result was obtained by Bahri and Coron [5], showing that if some homology group of \( D \) with coefficients in \( \mathbb{Z}_2 \) is not trivial, then (1) has at least one positive solution. See also [4, 6, 11, 18, 20, 32] for related results.

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