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SIGN CHANGING SOLUTIONS TO A BAHRI-CORON'S PROBLEM IN PIERCED DOMAINS

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Abstract. We consider the problem

$$\begin{cases} -\Delta u = |u|^{\frac{4}{N-2}} u & \text{in } \Omega \setminus \{B(\xi_1,\varepsilon) \cup B(\xi_2,\varepsilon)\}, \\ u = 0 & \text{on } \partial\left(\Omega \setminus \{B(\xi_1,\varepsilon) \cup B(\xi_2,\varepsilon)\}\right), \end{cases}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$, $N \geq 3$, ξ_1, ξ_2 are different points in Ω and ε is a small positive parameter. We show that, for ε small enough, the equation has at least one pair of sign changing solutions, whose positive and negative parts concentrate at ξ_1 and ξ_2 as ε goes to zero.

1. **Introduction.** Let D be a smooth bounded domain in $\mathbb{R}^N$, $N \geq 3$. Consider the following nonlinear elliptic problem

$$\Delta u + |u|^{\frac{4}{N-2}} u = 0 \quad \text{in} \quad D, \quad u = 0 \quad \text{on} \quad \partial D. \tag{1}$$

It is well known that the Sobolev embedding $H_0^1(D) \hookrightarrow L^{\frac{2N}{N-2}}(D)$ is not compact and that this lack of compactness makes the question of solvability of (1) quite delicate.

Pohozaev's identity [31] shows that problem (1) has only the trivial solution if the domain D is assumed to be strictly starshaped. On the other hand, Kazdan and Warner showed in [23] that if D is an annulus then (1) has a (unique) positive solution in the class of functions with radial symmetry. In [7], the authors study the asymptotic behavior of this solution as the radius of the inner ball of the annulus tends to zero. In the nonsymmetric case, Coron [17] found via variational methods that (1) is solvable and that it admits a positive solution under the assumption that D is a domain exhibiting a small hole. Substantial improvement of this result was obtained by Bahri and Coron [5], showing that if some homology group of D with coefficients in $\mathbf{Z}_2$ is not trivial, then (1) has at least one positive solution. See also [4, 6, 11, 18, 20, 32] for related results.

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